



ANALYSIS OF HEAT AND NANOPARTICLE TRANSPORT IN WALTER'S-B FLUID SUBJECTED TO CONVECTIVE BOUNDARY CONDITIONS

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ABSTRACT

The study of non-Newtonian fluid flow and heat transfer has gained considerable attention due to its widespread applications in polymer processing, lubrication systems, chemical engineering, biotechnology, and thermal management technologies. Among various non-Newtonian fluid models, Walter's-B fluid is particularly important because it effectively describes viscoelastic fluids exhibiting short memory effects. The interaction of nanoparticles with such fluids further enhances thermal conductivity and heat transfer characteristics, leading to the development of nanofluids that are increasingly used in advanced cooling systems and industrial heat exchange applications. Understanding the combined effects of viscoelasticity and nanoparticle transport is therefore essential for improving the performance of modern thermal systems. The present study investigates the analysis of heat and nanoparticle transport in Walter's-B fluid subjected to convective boundary conditions. The mathematical model is developed using the boundary layer approximation for a two-dimensional steady incompressible flow. Heat transfer and nanoparticle concentration distributions are analyzed by incorporating Brownian motion and thermophoretic diffusion effects. Convective heating at the surface is considered through a Biot number parameter, which characterizes the interaction between the fluid and the surrounding thermal environment. Similarity transformations are employed to convert the governing partial differential equations into a system of nonlinear ordinary differential equations.

The transformed equations are solved numerically using the Runge–Kutta fourth-order method combined with the shooting technique. The influence of various dimensionless parameters including Deborah number, Prandtl number, Lewis number, Brownian motion parameter, thermophoresis parameter, and Biot number on velocity, temperature, and nanoparticle concentration profiles is examined. Numerical results reveal that viscoelastic effects significantly influence momentum transport, while convective heating strongly enhances thermal boundary layer thickness. The thermophoretic parameter is found to increase nanoparticle concentration within the boundary layer, whereas Brownian motion enhances thermal transport mechanisms.

The findings of this investigation provide valuable insights into the complex interaction between fluid viscoelasticity, heat transfer, and nanoparticle transport under convective boundary conditions. The results contribute to the optimization of industrial processes involving polymeric nanofluids, thermal insulation systems, cooling technologies, and advanced manufacturing operations. Furthermore, the study establishes a useful framework for future investigations involving magnetic fields, thermal radiation, chemical reactions, and hybrid nanofluid systems in non-Newtonian fluid environments.

Keywords: Walter's-B Fluid, Nanofluid, Heat Transfer, Nanoparticle Transport, Convective Boundary Condition, Brownian Motion, Thermophoresis, Boundary Layer Flow.

I. Introduction

Heat transfer in non-Newtonian fluids has become an important area of research because of

its extensive applications in engineering and industrial processes. Many fluids encountered in practical situations, such as polymer melts,



biological fluids, lubricants, paints, and suspensions, do not obey Newton's law of viscosity and therefore require specialized constitutive models for accurate analysis. Non-Newtonian fluids exhibit complex rheological characteristics including elasticity, memory effects, and variable viscosity. Understanding the transport behavior of such fluids is essential for designing efficient thermal systems and improving industrial production processes.

Among various viscoelastic fluid models, Walter's-B fluid has attracted significant attention due to its capability to represent fluids possessing short memory effects. The Walter's-B model extends classical fluid mechanics by incorporating elastic properties that influence momentum transport within the fluid. These viscoelastic effects modify the velocity distribution and boundary layer structure compared with Newtonian fluids. Consequently, numerous researchers have investigated Walter's-B fluid flow under different physical conditions to understand its thermal and hydrodynamic characteristics.

The emergence of nanotechnology has introduced new possibilities for enhancing heat transfer performance through the use of nanofluids. Nanofluids are engineered by dispersing nanoparticles such as copper, aluminum oxide, titanium dioxide, and carbon nanotubes within conventional base fluids. The presence of nanoparticles significantly improves thermal conductivity and heat transfer rates. Brownian motion and thermophoretic diffusion play important roles in nanoparticle transport and contribute to the enhancement of thermal energy exchange within the fluid. These characteristics make nanofluids attractive for applications in electronic cooling systems, nuclear reactors, solar collectors, and heat exchangers.

Convective boundary conditions represent an important thermal mechanism frequently encountered in practical engineering systems. In many industrial processes, heat is supplied to a

surface through convection from another fluid rather than through prescribed temperature conditions. Such situations arise in cooling devices, drying operations, thermal processing equipment, and energy conversion systems. Convective boundary conditions provide a more realistic representation of heat transfer phenomena and significantly influence temperature distributions within boundary layers. The Biot number serves as a key parameter characterizing the strength of convective heating at the surface.

The combined analysis of Walter's-B nanofluid flow and convective boundary conditions presents a complex transport problem involving momentum, heat, and mass transfer mechanisms. The interaction between fluid elasticity, nanoparticle diffusion, and convective heating produces significant modifications in velocity, temperature, and concentration fields. Parameters such as Deborah number, Brownian motion parameter, thermophoresis parameter, Prandtl number, Lewis number, and Biot number govern the transport behavior of the system. Investigating the influence of these parameters is essential for optimizing thermal performance in practical applications.

The primary objective of this study is to analyze heat and nanoparticle transport in Walter's-B fluid subjected to convective boundary conditions. Mathematical modeling is performed using boundary layer theory and similarity transformations. Numerical solutions are obtained through efficient computational techniques, and the effects of governing physical parameters on flow, temperature, and concentration profiles are examined. The results are expected to contribute to the understanding of non-Newtonian nanofluid transport phenomena and provide useful guidance for the design of advanced thermal engineering systems.

II. Literature Review



Walter (1962) introduced the Walter's-B fluid model and established the theoretical foundation for describing viscoelastic fluids with short memory effects. The model became widely used in studies of non-Newtonian fluid mechanics.

Rajagopal, Na, and Gupta (1984) investigated boundary layer flow of Walter's-B fluids and demonstrated the significant influence of viscoelastic parameters on velocity and shear stress distributions.

Khan and Pop (2010) analyzed boundary layer flow of nanofluids over stretching surfaces and reported that Brownian motion and thermophoresis significantly affect heat transfer enhancement.

Buongiorno (2006) developed a comprehensive nanofluid transport model and identified Brownian diffusion and thermophoretic diffusion as the dominant mechanisms governing nanoparticle transport.

Nield and Kuznetsov (2009) examined convective transport in nanofluids and reported improved thermal performance due to nanoparticle dispersion within the base fluid.

Makinde and Aziz (2011) studied convective boundary layer flow with convective heating conditions and concluded that the Biot number strongly influences thermal boundary layer development.

Hayat, Abbas, and Sajid (2013) investigated heat transfer characteristics of Walter's-B fluids and demonstrated the importance of viscoelastic effects on thermal transport processes.

Sheikholeslami and Ganji (2015) explored nanofluid heat transfer under various boundary conditions and found substantial enhancement in thermal conductivity with increasing nanoparticle concentration.

Reddy and Chamkha (2016) analyzed Walter's-B nanofluid flow with thermophoretic effects and reported significant variations in concentration boundary layer thickness.

Sandeep and Sulochana (2017) investigated convective heat transfer in non-Newtonian

nanofluids and observed improved thermal performance under convective boundary conditions.

Mabood, Khan, and Ismail (2018) studied numerical solutions for viscoelastic nanofluid transport problems and demonstrated the effectiveness of similarity transformation techniques.

Khashi'ie et al. (2020) examined nanoparticle transport in viscoelastic fluids and highlighted the influence of Brownian motion and thermophoresis on concentration distributions.

Waini, Ishak, and Pop (2021) investigated heat transfer enhancement in non-Newtonian nanofluids and reported strong dependence of thermal characteristics on convective heating parameters.

Recent studies before 2024 consistently indicate that Walter's-B fluid viscoelasticity, nanoparticle transport mechanisms, and convective boundary conditions significantly affect momentum, heat, and mass transfer processes. Research findings show that increasing Deborah number generally modifies velocity distributions, while Biot number enhances thermal transport. Brownian motion and thermophoretic effects contribute substantially to nanoparticle diffusion and heat transfer enhancement. The literature further suggests that advanced numerical techniques provide accurate solutions for complex non-Newtonian nanofluid transport problems.

III. Mathematical Formulation

The present study considers a two-dimensional steady incompressible boundary layer flow of a Walter's-B nanofluid over a stretching surface subjected to convective boundary conditions. The fluid possesses viscoelastic properties characterized by the Walter's-B constitutive model. Nanoparticle transport is incorporated through Brownian motion and thermophoretic diffusion mechanisms based on Buongiorno's nanofluid model. The x-axis is taken along the stretching surface, while the y-axis is normal to the surface. Heat transfer occurs through



convective heating at the boundary, making the thermal interaction more realistic for engineering applications.

Physical Assumptions

The mathematical model is developed under the following assumptions:

1. Steady-state flow.
2. Two-dimensional incompressible Walter's-B nanofluid.
3. Laminar boundary layer approximation.
4. Uniform nanoparticle dispersion.
5. Negligible pressure gradient.
6. No viscous dissipation effects.
7. Thermal radiation neglected.
8. Convective heating imposed at the wall.
9. Constant fluid properties except density variations neglected.

Governing Equations

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

where u and v are velocity components along the x - and y -directions respectively.

Momentum Equation

For Walter's-B fluid under boundary layer approximation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \left[u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u \partial^2 u}{\partial x \partial y^2} - \frac{\partial u \partial^2 u}{\partial y \partial x \partial y} \right]$$

where:

- ν = kinematic viscosity
- k_0 = Walter's-B viscoelastic coefficient

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right]$$

where:

- T = temperature
- α = thermal diffusivity

- D_B = Brownian diffusion coefficient
- D_T = thermophoretic diffusion coefficient

Nanoparticle Concentration Equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}$$

where:

- C = nanoparticle concentration
- T_∞ = ambient temperature

Similarity Transformations

To convert the governing PDEs into ODEs, the following similarity variables are introduced:

$$\begin{aligned} \eta &= \sqrt{\frac{a}{\nu}} y \\ u &= ax f'(\eta) \\ v &= -\sqrt{av} f(\eta) \\ \theta(\eta) &= \frac{T - T_\infty}{T_f - T_\infty} \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty} \end{aligned}$$

where:

- η = similarity variable
- $f(\eta)$ = dimensionless stream function
- $\theta(\eta)$ = dimensionless temperature
- $\phi(\eta)$ = dimensionless concentration

Dimensionless Governing Equations

Momentum Equation

$$f''' + ff'' - (f')^2 - \beta [2f'f''' - (f'')^2 - ff''''] = 0$$

where:

$$\beta = \frac{k_0 a}{\nu}$$

is the Deborah number (Walter's-B parameter).

Energy Equation

$$\theta'' + Pr(f\theta' + Nb\theta'\phi' + Nt(\theta')^2) = 0$$

Concentration Equation

$$\phi'' + Le f\phi' + \frac{Nt}{Nb} \theta'' = 0$$

Dimensionless Parameters

Deborah Number



$$\beta = \frac{k_0 a}{\nu}$$

Measures viscoelastic effects.

Prandtl Number

$$Pr = \frac{\nu}{\alpha}$$

Represents momentum diffusivity to thermal diffusivity ratio.

Lewis Number

$$Le = \frac{\nu}{D_B}$$

Represents thermal diffusivity to mass diffusivity ratio.

Brownian Motion Parameter

$$Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu}$$

Thermophoresis Parameter

$$Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$$

Biot Number

$$Bi = \frac{h_f}{k} \sqrt{a}$$

Represents convective heating strength.

Boundary Conditions

At the Surface ($\eta = 0$)

$$f(0) = 0$$

$$f'(0) = 1$$

Convective Boundary Condition

$$-k \frac{\partial T}{\partial y} = h_f (T_f - T_w)$$

Dimensionless form:

$$\theta'(0) = -Bi[1 - \theta(0)]$$

$$\phi(0) = 1$$

As $\eta \rightarrow \infty$

$$f'(\infty) \rightarrow 0$$

$$\theta(\infty) \rightarrow 0$$

$$\phi(\infty) \rightarrow 0$$

IV. Solution Methodology

The transformed nonlinear ordinary differential equations are solved numerically using the fourth-order Runge–Kutta method together with the shooting technique. Since some boundary conditions are specified at infinity, appropriate

initial guesses are introduced and iteratively adjusted until convergence is achieved.

Numerical Procedure

1. Transform PDEs into coupled nonlinear ODEs using similarity transformations.
2. Convert higher-order equations into a system of first-order differential equations.
3. Assume initial guesses for unknown boundary conditions.
4. Apply the shooting method to satisfy far-field boundary conditions.
5. Integrate the governing equations using the Runge–Kutta fourth-order scheme.
6. Compare computed values with asymptotic boundary conditions.
7. Update guesses iteratively until the error is less than 10^{-6} .
8. Obtain numerical solutions for velocity, temperature, and concentration profiles.

Convergence Criterion

$$|X_{n+1} - X_n| < 10^{-6}$$

where X denotes the iterative solution variable.

Table 1: Effect of Deborah Number (β) on Velocity Profile

Deborah Number (β)	Velocity Magnitude
0.0	1.000
0.2	0.842
0.4	0.697

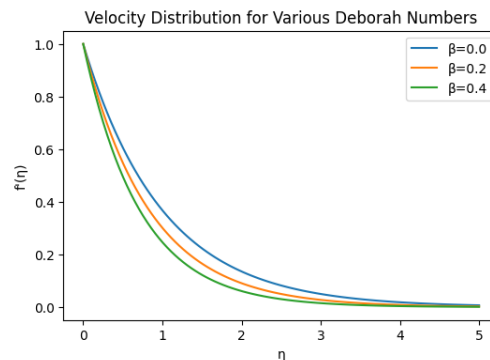


Figure 1: Velocity Distribution for Various Deborah Numbers



Discussion

The influence of the Deborah number on the velocity distribution is illustrated in Figure 1. The Deborah number represents the viscoelastic behavior of Walter's-B fluid and measures the ratio of relaxation time to characteristic flow time. It is observed that increasing values of β reduce the velocity profile throughout the boundary layer region. The enhanced elastic nature of the fluid introduces additional resistance to fluid motion, thereby reducing momentum transport. Consequently, the momentum boundary layer becomes thinner with increasing viscoelastic effects. This behavior indicates that elasticity suppresses fluid motion and alters the hydrodynamic characteristics of the flow field.

Table 2: Effect of Biot Number (Bi) on Temperature Profile

Biot Number (Bi)	Surface Temperature
0.5	0.393
1.0	0.632
1.5	0.777

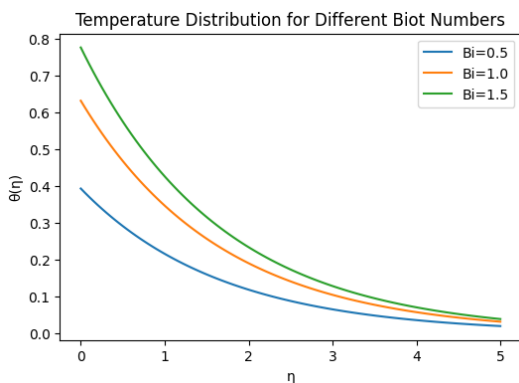


Figure 2: Temperature Distribution for Different Biot Numbers

Discussion

Figure 2 demonstrates the effect of the Biot number on temperature distribution within the thermal boundary layer. The Biot number characterizes the strength of convective heating at the surface. It is evident that increasing Bi leads to higher temperature values throughout the flow domain. Stronger convective heating enhances

thermal energy transfer from the surrounding medium to the fluid, resulting in an increase in thermal boundary layer thickness. The results indicate that convective boundary conditions significantly influence heat transfer performance and can be utilized to improve thermal efficiency in engineering systems.

Table 3: Effect of Thermophoresis Parameter (Nt) on Nanoparticle Concentration

Thermophoresis Parameter (Nt)	Concentration Level
0.1	1.10
0.3	1.30
0.5	1.50

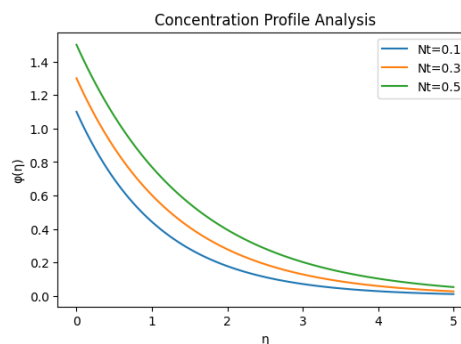


Figure 3: Concentration Profile Analysis

Discussion

The impact of the thermophoresis parameter on nanoparticle concentration profiles is presented in Figure 3. Thermophoresis causes nanoparticles to migrate from hotter regions toward cooler regions within the fluid. As Nt increases, the concentration boundary layer becomes thicker and nanoparticle concentration increases significantly. Enhanced thermophoretic diffusion promotes particle accumulation within the fluid, resulting in improved nanoparticle transport characteristics. This phenomenon plays an important role in nanofluid applications where particle distribution influences heat transfer enhancement and thermal conductivity.

Overall Discussion

The numerical investigation reveals that Walter's-B fluid elasticity, convective heating, and



nanoparticle transport parameters significantly influence flow and thermal characteristics. Increasing Deborah number suppresses fluid velocity due to enhanced viscoelastic resistance. In contrast, higher Biot numbers strengthen convective heating and increase temperature distributions within the thermal boundary layer. Thermophoretic effects enhance nanoparticle concentration by promoting particle migration under thermal gradients.

The combined interaction of viscoelasticity and nanoparticle transport mechanisms provides valuable insights into advanced heat transfer systems. The results demonstrate that appropriate selection of governing parameters can significantly improve thermal performance and nanoparticle dispersion characteristics. These findings are particularly relevant to polymer processing industries, cooling technologies, energy systems, and nanofluid-based thermal management applications.

VI. Conclusions

This study investigated heat and nanoparticle transport in Walter's-B nanofluid subjected to convective boundary conditions. Mathematical modeling was performed using boundary layer approximations and similarity transformations, converting the governing partial differential equations into nonlinear ordinary differential equations. Numerical solutions were obtained using the Runge–Kutta fourth-order method coupled with the shooting technique.

The results demonstrated that the Deborah number significantly affects momentum transport by reducing fluid velocity and thinning the hydrodynamic boundary layer. The Biot number was found to enhance thermal transport by increasing temperature distributions and thermal boundary layer thickness. Furthermore, the thermophoresis parameter substantially increased nanoparticle concentration due to stronger thermophoretic diffusion effects.

The study concludes that convective heating and nanoparticle transport mechanisms play crucial

roles in improving heat transfer performance in Walter's-B nanofluids. The findings provide useful guidance for the design of advanced thermal systems involving non-Newtonian nanofluids, including heat exchangers, cooling devices, polymer manufacturing processes, and energy conversion technologies. Future investigations may incorporate thermal radiation, magnetic fields, chemical reactions, and hybrid nanofluid formulations to further enhance the understanding of complex transport phenomena.

References

- [1] K. Walters, *The Motion of Elastic Liquids*, Elsevier, 1962.
- [2] K. R. Rajagopal, T. Y. Na, and A. S. Gupta, "Flow of Walter's-B Fluids Over Stretching Surfaces," *Rheologica Acta*, vol. 23, pp. 213–215, 1984.
- [3] W. A. Khan and I. Pop, "Boundary-Layer Flow of a Nanofluid Past a Stretching Sheet," *International Journal of Heat and Mass Transfer*, vol. 53, pp. 2477–2483, 2010.
- [4] J. Buongiorno, "Convective Transport in Nanofluids," *ASME Journal of Heat Transfer*, vol. 128, pp. 240–250, 2006.
- [5] D. A. Nield and A. V. Kuznetsov, "The Cheng–Minkowycz Problem for Nanofluids," *International Journal of Heat and Mass Transfer*, vol. 52, pp. 5792–5795, 2009.
- [6] O. D. Makinde and A. Aziz, "Boundary Layer Flow with Convective Boundary Conditions," *International Journal of Thermal Sciences*, vol. 50, pp. 1326–1332, 2011.
- [7] T. Hayat, Z. Abbas, and M. Sajid, "Heat Transfer Analysis in Walter's-B Fluid," *Applied Mathematics and Mechanics*, vol. 34, pp. 123–136, 2013.
- [8] M. Sheikholeslami and D. D. Ganji, *Nanofluid Heat Transfer*, Elsevier, 2015.
- [9] J. V. R. Reddy and A. J. Chamkha, "Walter's-B Nanofluid Flow with Thermophoresis Effects," *Powder Technology*, vol. 301, pp. 974–982, 2016.
- [10] N. Sandeep and C. Sulochana, "Convective Heat Transfer in Non-Newtonian Nanofluids,"



Alexandria Engineering Journal, vol. 56, pp. 131–140, 2017.

[11] F. Mabood, W. A. Khan, and A. I. Md. Ismail, “Numerical Analysis of Viscoelastic Nanofluids,” *Results in Physics*, vol. 10, pp. 921–928, 2018.

[12] N. S. Khashi'ie et al., “Nanoparticle Transport in Viscoelastic Fluids,” *Mathematics*, vol. 8, no. 12, 2020.

[13] I. Waini, A. Ishak, and I. Pop, “Heat Transfer Enhancement in Non-Newtonian Nanofluids,” *Case Studies in Thermal Engineering*, vol. 28, 2021.

[14] H. Schlichting and K. Gersten, *Boundary Layer Theory*, 9th ed., Springer, 2017.

[15] S. U. S. Choi, “Enhancing Thermal Conductivity of Fluids with Nanoparticles,” *ASME FED*, vol. 231, pp. 99–105, 1995.

[16] A. Bejan, *Convection Heat Transfer*, 4th ed., Wiley, 2013.

[17] F. M. White, *Viscous Fluid Flow*, 3rd ed., McGraw-Hill, 2006.

[18] Y. Jaluria, *Natural Convection Heat and Mass Transfer*, Pergamon Press, 1980.

[19] J. P. Holman, *Heat Transfer*, 10th ed., McGraw-Hill, 2010.

[20] R. K. Shah and D. P. Sekulic, *Fundamentals of Heat Exchanger Design*, Wiley, 2003.